Counting, Permutations, and Combinations

- Topics: Permutations and Combinations

- Objective: Students will be able to calculate how many permutations or combinations can be made given a specific situation and find the probability of that event happening.

- Standards: CCSS Math: 7.SP.C.8, HSS.CP.B.9

The Counting Principle

Definition: The Fundamental Counting Principle (also called the counting rule) is a way to figure out the number of outcomes in a probability problem. Basically, you multiply the events together to get the total number of outcomes.

Example: Oula is going on an outdoor expedition with his family. The expedition will include a hunting event, a fishing event, a hiking event, and a camping event. There are 4 hunting, 7 fishing, 6 hiking, and 3 camping events for Oula's family to choose from.

How many different outdoor expeditions are possible?

$4 \times 7 \times 6 \times 3 =$

Permutations and Combinations

Definition: In mathematics, **permutation** is the act of arranging the members of a set into a sequence or order, or, if the set is already ordered, rearranging its elements—a process called permuting.

Permutations differ from **combinations,** which are selections of some members of a set regardless of order.

→ groups

Factorial (!): In mathematics, the factorial of a positive integer n, denoted by n!, is the product of all positive integers less than or equal to n.

Example: 4! = 4*3*2*1 = 24

$4! = 4 \times 3 \times 2 \times 1$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

Permutations and Combinations

Formulas:

How many you have.

How many you're using.

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

5 3

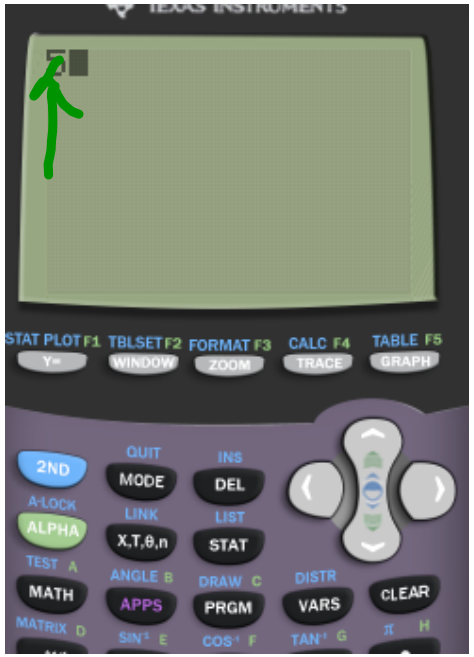
$\frac{5!}{(5-3)!}$

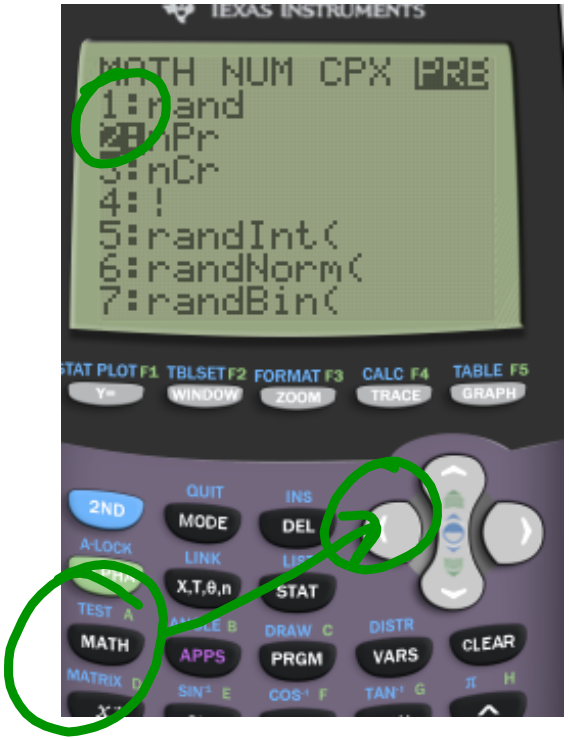
2.

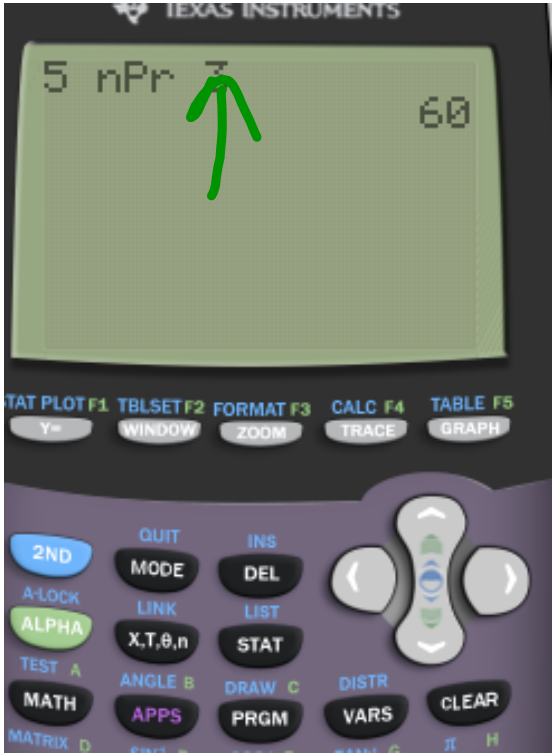
$$\frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

60

Calculator Method:







How many numbers between 1 and 100 (inclusive) are divisible by 3 or 2?

$3 \times 2 = 6$

$\frac{100}{3} = 33$

$\frac{100}{6} = 16$

$\frac{100}{2} = 50$

$33 + 50 - 16 = 67$

apple = $\frac{5!}{2!}$ Total letters / Repeated letters

$\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60$ ☺

Friends or Fighting?

5

You need to put your reindeer, Rudy, Ezekiel, Gloopin, Bloopin, and Prancer, in a single-file line to pull your sleigh. However, Bloopin and Rudy are fighting, so you have to keep them apart, or they won't fly.

How many ways can you arrange your reindeer?

Friends: $5-1=4 \Rightarrow 4! \times 2$

$(4 \times 3 \times 2 \times 1) \times 2 = 48$ Friends

Fighting: All arrang − friends

$5P5 - (4P4) \times 2$

$120 - 48 = 72$

Probability with Permutations and Combinations

Probabilities will always be a fraction:

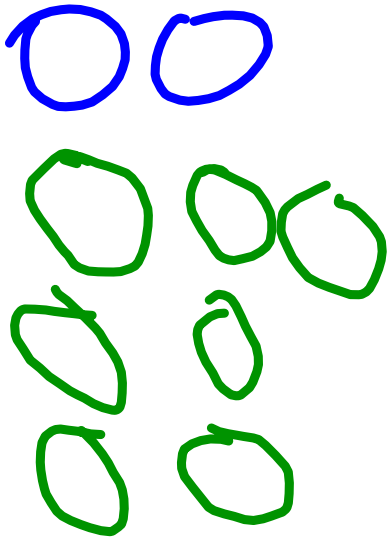
$$\frac{\text{What You Want}}{\text{Total Number of Outcomes}}$$

Probability with Permutations and Combinations

$$\frac{\text{What You Want}}{\text{Total Number of Outcomes}}$$

There are 9 students in a class: 2 boys and 7 girls.

If the teacher picks a group of 4 at random, what is the probability that everyone in the group is a girl?



How many want → $_7C_4$ ← How many in group

$$\frac{{}_7C_4}{{}_9C_4}$$

If you flip a fair coin 7 times, what is the probability that you will get exactly 2 tails?

$_7C_2$

$\frac{{}_7C_2}{2^7}$

7 coins → # of outcomes

2^7

H T

128

Displaying and Comparing Quantitative Data

You should be working on the following skills:

1. The counting principle
2. Permutations
3. Combinations
4. Permutations and combinations
5. Probability of permutations and combinations

Attachments

Ztable.pdf